

## CHAPTER 4 : TIME VALUE OF MONEY

At the foundation level with regards to Business Mathematics the topic Time Value of money is very important for students not only to acquire professional knowledge but also for examination point of view. Here in this capsule an attempt is made for solving and understanding the concepts of time value of money with the help of following questions with solutions.

## Problems on Simple, Compound and Effective rate of interest

Simple interest = P.T.I., Where P = Principal, T = Time, I = Rate of Interest

1. A sum of money amount to ₹ 6,200 in 2 years and ₹ 7,400 in 3 years. The principal and rate of interest is

**Solution:** A sum of money in 2 years =  $P + P.2.I = ₹ 6200$

A sum of money in 3 years =  $P + P.3.I = ₹ 7400$

Interest in 1 year = ₹ 1200; Interest in 2 years = ₹ 2400

Amount = ₹ 6200, P = Principal =  $6200 - 2400 = ₹ 3800$ .

$$2400 = 3800 \times 2 \times \frac{I}{100}, I = \text{rate of interest} = 31.58\%$$

2. A sum of money doubles itself in 10 years. The number of years it would triple itself is

**Solution:**  $2P = P + \frac{PTR}{100}$ ,  $P = \frac{PTR}{PTR/100}$ ; T = 10 then R = 10%

$$3P = P + \frac{PTR}{100} \text{ then } 2P = \frac{PTR}{100} \text{ and } R = 10\%$$

Time (T) = 20 years.

Amount =  $P(1+i)^n$

Compound rate of interest =  $P(1+i)^n - P = A - P$

where P = principle i = interest n = conversion period

3. The population of a town increases every year by 2% of the population at the beginning of that year. The number of years by which the total increase of population be 40% is

**Solution:**  $1.4P = P(1+0.02)^n$

$$(1.02)^n = 1.4, n = 17 \text{ years (app)}$$

Depreciation (A) =  $P(1-i)^n$

Where, A = Scrap Value, P = Original Cost, I = Depreciated at the rate, n = Number of years

4. A machine is depreciated at the rate of 20% on reducing balance. The original cost of the machine was ₹ 1,00,000 and its ultimate scrap value was ₹ 30,000. The effective life of the machine is

**Solution:** Here A = Scrap Value = 30,000 and

P = Original Cost = ₹ 1,00,000

$$30000 = 100000(1-0.2)^n$$

$$3/10 = 0.3 = (0.8)^n, n = 5.4 \text{ years}$$

5. The useful life of a machine is estimated to be 10 years and cost ₹ 10,000. Rate of depreciation is 10% p.a. The scrap value at the end of its life is

**Solution:** Here A = Scrap Value = ?,

P = Original cost = 10,000 n = 10, I = 10%

$$A = 100000(1-0.1)^{10} = 10000(0.9)^{10}$$

$$A = ₹ 3486.78$$

Effective rate of interest =  $(1+i)^n - 1$

6. The effective rate of interest corresponding a nominal rate of 7% p.a convertible quarterly is

**Solution:** Effective rate of interest =  $(1+i)^n - 1$ , here n = 4,

$$i = 0.07/4 = 0.0175$$

$$= (1+0.0175)^4 - 1 = 1.07186 - 1 = 7.19\%$$

The difference between simple and compound interest for 2 years =  $P.i^2$ , where P = Principal, i = interest

7. The difference between the S.I and the C.I on ₹ 2,400 for 2 years at 5% p.a is

**Solution:** The difference between simple and compound interest for 2 years =  $2400(0.05)^2 = ₹ 6$ .

The difference between simple and compound interest for 3 years =  $3P.i^2 + P.i^3$ , where P = Principal i = Interest

8. The difference between the S.I and the C.I on a certain money invested for 3 years at 6% p.a is ₹ 110.16 the principle is

**Solution:** The differences between simple and compound interest for 3 years

$$= 110.16 = P(3i^2 + i^3) = P(3 \times 0.06^2 + 0.06^3)$$

$$110.16 = P(0.0108 + 0.000216) = P(0.011016)$$

$$P = \frac{110.16}{0.011016} = ₹ 10,000$$

9. The annual birth and death rates per 1,000 are 39.4 and 19.4 respectively. The number of years in which the population will be doubled assuming there is no immigration or emigration is

**Solution:** Here given, birth rate per 1,000 = 39.4 and death rates per 1,000 = 19.4

difference = 20% per 1000 population

$$\text{growth rate} = \frac{20}{1000} \times 100 = 2\%$$

Future population double

$$P = 1000, A = 2000, r = 2\%$$

$$2000 = 1000(1+0.02)^n$$

$$(1.02)^n = 2$$

Number of years = n = 35

10. What annual rate of interest compounded annually doubles an investment in 7 years?

(Given that  $2^{1/7} = 1.104090$ )

**Solution:** If the principal be P,  $A_n = 2P$

$$\text{Since } A_n = P(1+i)^n$$

$$2P = P(1+i)^7$$

$$2^{1/7} = (1+i)$$

$$1.104090 = 1+i$$

$$i = 0.10409, \text{ Required rate of interest} =$$

$$10.41\% \text{ per annum}$$

11. Vidya deposited ₹ 60000 in a bank for two years with the interest rate of 5.5% p.a. How much interest she would earn? what will be the final value of investment?

**Solution:** Required interest amount is given by,

$$I = P \times it = ₹ 60,000 \times \frac{5.5}{100} \times 2 = ₹ 6,600$$

The amount value of investment is given by,  $A = P + I = ₹ (60,000 + 6600) = ₹ 66,600$

12. Rajiv invested ₹ 75,000 in a bank at the rate of 8% p.a. simple interest rate. He received ₹ 135,000 after the end of term. Find out the period for which sum was invested by Rajiv.

**Solution:** We know  $A = P + Pit = P(1+it)$

$$\text{i.e. } 135000 = 75000 \left(1 + \frac{8}{100} \times t\right)$$

$$\frac{135000}{75000} = \frac{100+8t}{100}$$

$$1.8 \times 100 - 100 = 8t$$

$$80 = 8t$$

$$t (\text{Time}) = 10 \text{ years}$$

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13. Which is a better investment, 3.6% per year compounded monthly or 3.2% per year simple interest? Given that  $(1+0.003)^{12} = 1.0366$ .

Effective rate of interest  $E = (1+i)^n - i$

**Solution:**  $i = 3.6/12 = 0.3\% = 0.003, n = 12$   
 $E = (1+i)^n - 1 = (1+0.003)^{12} - 1 = 1.0366 - 1 = 0.0366$  or  $= 3.66\%$

Effective rate of interest (E) 3.66% is more than the simple interest so the Effective rate of interest (E) is better investment than the simple interest 3.2% per year.

14. The C.I on ₹ 4,000 for 6 months at 12% p.a payable quarterly is

**Solution:** Here  $P = ₹ 4000, n = 6/3 = 2, r = 0.12/4 = 0.03$   
 Compound Interest  $= [P(1+i)^n - P] = [4000(1+0.03)^2 - 4000] = ₹ 243.60$

**Annuity applications**

$F =$  Future value  $= C.F. (1+i)^n$  Where C.F = Cash flow  
 $i =$  rate of interest,  $n =$  time period

15. Ravi invest ₹ 5000 in a two-year investment that pays you 12% per annum. Calculate the future value of the investment

**Solution:** We know,  $F =$  Future value  $= C.F.(1+i)^n$ , Where C.F= Cash flow  $= ₹ 5000, i =$  rate of interest  $= 0.12, n =$  time period  $= 2$   
 $F = ₹ 5000(1+0.12)^2 = ₹ 5000 \times 1.2544 = ₹ 6272$ .

Annuity regular means :First payment at the end of the period.  
 Future value of the annuity regular  $= A(n,i) = A \cdot \left[ \frac{(1+i)^n - 1}{i} \right]$

**Annuity regular:** In annuity regular first payment/receipt takes place at the end of first period.

16. Find the future value of an annuity of ₹ 5000 is made annually for 7 years at interest rate of 14% compounded annually. [Given that  $(1.14)^7 = 2.5023$ ]

**Solution:** Here annual payment  $A = ₹ 5000, n = 7, i = 14\% = 0.14$

$$\text{Future value of the annuity} = A(7, 0.14) = 5000 \cdot \left[ \frac{(1+0.14)^7 - 1}{0.14} \right] = 5000 \left[ \frac{(2.5023 - 1)}{0.14} \right] = ₹ 53653.57$$

**Future value of Annuity due or Annuity Immediate:** When the first receipt or payment at the beginning of the annuity) it is called annuity due or annuity immediate.

17. ₹ 2000 is invested at the end of each month in an account paying interest 6% per year compounded monthly. What is the future value of this annuity after 10<sup>th</sup> payment? Given that  $(1.005)^{10} = 1.0511$

**Solution:** Here  $A = ₹ 2000, n = 10, i = 6\%$  per annum  $= 6/12\%$  per month  $= 0.005$

Future value of annuity after 10 months is given by

$$A(n,i) = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$A(10, 0.005) = 2000 \left[ \frac{(1+0.005)^{10} - 1}{0.005} \right] = 2000 \left[ \frac{(1.0511) - 1}{0.005} \right]$$

$$= 2000 \times 10.22 = ₹ 20440$$

Future value of the annuity regular or annuity

$$\text{due} = A \left[ \frac{(1+i)^n - 1}{i} \right] \times (1+i)$$

18. Swati invests ₹ 20,000 every year starting from today for next 10 years. Suppose interest rate 8% per annum compounded annually. Calculate future value of the annuity. Given that  $(1 + 0.08)^{10} = 2.158925$

**Solution:** Calculate future value as though it were an ordinary annuity. Future value of the annuity as if it were an ordinary annuity

$$= ₹ 20000 \left[ \frac{(1+0.08)^{10} - 1}{0.08} \right]$$

$$= ₹ 20000 \times 14.486563 = ₹ 289731.25$$

Multiply the result by  $(1+i) = ₹ 289731.25 \times (1+0.08) = ₹ 312909.76$

19. What is the present value of ₹ 100 to be received after two years compounded annually at 10%.

**Solution:** Here  $A_n = ₹ 100, i = 10\% = 0.1, n = 2$

$$\text{Required present value} = \frac{A_n}{(1+i)^n} = \frac{100}{(1+0.1)^2} = \frac{100}{(1.21)} = 82.64$$

Thus ₹ 82.64 shall grow to ₹ 100 after 2 years at 10% compounded annually.

20. Find the present value of ₹ 10000 to be required after 5 years if the interest rate be 9%. Given that  $(1.09)^5 = 1.5386$ .

**Solution:** Here  $i = 0.09, n = 5, A_n = 10000$

$$\text{Required present value} = \frac{A_n}{(1+i)^n} = \frac{10000}{(1+0.09)^5} = \frac{10000}{(1.5386)} = ₹ 6499.42$$

$$\text{Present Value of Annuity regular} = A = P(n, i) = A \cdot \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right],$$

$$P(n,i) = \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

21. Soni borrows ₹ 5,00,000 to buy a car. If he pays equal instalments for 10 years and 10% interest on outstanding balance, what will be the equal annual instalment? Given  $[P(10,0.10) = 6.14457]$

**Solution:** We know,  $A = \frac{V}{P(n,i)}$  Here  $V = ₹ 500000, n = 10, I = 10\%$  p.a.  $= 0.10$

$$\text{Annual Instalment} = \frac{V}{P(n,i)} = ₹ \frac{5,00,000}{6.14457} = ₹ 8,1372.66$$

22. If ₹ 10,000 is paid every year for ten years to pay off a loan. What is the loan amount if interest rate be 14% per annum compounded annually? Given  $[P(10,0.14) = 5.21611]$

**Solution:**  $V = A \cdot P(n,i)$  Here  $A = ₹ 10000, n = 10, i = 0.14$   
 $V = 10000 \times P(10, 0.14)$   
 $= 10000 \times 5.21611 = ₹ 52161.10$   
 Therefore, the loan amount is ₹ 52161.10

23. Ram bought a Scooter costing ₹73000 by making a down payment of ₹ 3000 and agreeing to make equal annual payment for four years. How much would be each payment if the interest on unpaid amount be 14% compounded annually? Given  $[P(4, 0.14) = 2.91371]$

**Solution:** In the present case we have present value of the annuity i.e. ₹ 70000 (73000-3000) and we have to calculate equal annual payment over the period of four years.

We know that,  $V = A \cdot P(n, i)$  Here  $n = 4$  and  $I = 0.14$

$$A = \frac{V}{P(n,i)} = \frac{70000}{P(4,0.14)} = \frac{70000}{2.91371}$$

Therefore, each payment = ₹ 24024.35

24. Suppose your Father decides to gift you ₹ 20,000 every year starting from today for the next six years. You deposit this amount in a bank as and when you receive and get 10% per annum interest rate compounded annually. What is the present value of this annuity?

**Solution:** For calculating present value of the annuity immediate following steps will be followed. Present value of the annuity as if it were a

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regular annuity for one year less i.e. for five years

$$= ₹ 20,000 \times P(5, 0.10)$$

$$= ₹ 20,000 \times 3.79079 = ₹ 75815.80$$

Add initial cash deposit to the value, ₹ (75815.80+20,000) = ₹ 95815.80

**Sinking Fund:** Interest is computed at end of every period with specified interest rate.

25. How much amount is required to be invested every year so as to accumulate ₹ 5,00,000 at the end of 10 years if interest is compounded annually at 10%? Given A. (10, 0.1) = 15.9374248

**Solution:** Here  $A = 500000, n = 10, A(n, i) = \left[ \frac{(1+i)^n - 1}{i} \right] =$   
 $\left[ \frac{(1+0.1)^{10} - 1}{0.1} \right] = 15.9374248$   
 since  $A = P \cdot A. (n, i)$   
 $500000 = P \cdot A. (10, 0.1) = P \times 15.9374248$   
 $P = \left[ \frac{500000}{15.9374248} \right] = ₹ 31372.70$

26. ABC Ltd. wants to lease out an asset costing ₹ 360000 for a five year period. It has fixed a rental of Rs.105000 per annum payable annually starting from the end of first year. Suppose rate of interest is 14% per annum compounded annually on which money can be invested by the company. Is this agreement favorable to the company?

**Solution:** First, we have to compute the present value of the annuity of ₹ 105000 for five years at the interest rate of 14% p.a. compounded annually.

The present value V of the annuity is given by  
 $V = A \cdot P(n, i) = 105000 \times P(5, 0.14)$   
 $= 105000 \times 3.43308 = ₹ 360473.40$

which is greater than the initial cost of the asset and consequently leasing is favourable to the lessor.

27. A company is considering proposal of purchasing a machine either by making full payment of ₹ 4000 or by leasing it for four years at an annual rate of ₹ 1250. Which course of action is preferable if the company can borrow money at 14% compounded annually?

**Solution:** The present value V of annuity is given by

$$V = A \cdot P(n, i) = 1250 \times P(4, 0.14)$$

$$= 1250 \times 2.91371 = ₹ 3642.11$$

which is less than the purchase price, and consequently leasing is preferable.

28. A machine can be purchased for ₹ 50000. Machine will contribute ₹ 12000 per year for the next five years. Assume borrowing cost is 10% per annum compounded annually. Determine whether machine should be purchased or not.

**Solution:** The present value of annual contribution

$$V = A \cdot P(n, i)$$

$$= 12000 P(5, 0.10) = 12000 \times 3.79079$$

$$= ₹ 45489.48$$

which is less than the initial cost of the machine. Therefore, machine must not be purchased.

29. A machine with useful life of seven years costs ₹ 10000 while another machine with useful life of five years costs ₹ 8000. The first machine saves labour expenses of ₹ 1900 annually and the second one saves labour expenses of ₹ 2200 annually. Determine the preferred course of action. Assume cost of borrowing as 10% compounded per annum.

**Solution:** The present value of annual cost savings for the first machine =  
 $₹ 1900 \cdot P(7, 0.10)$   
 $= ₹ 1900 \times 4.86842 = ₹ 9250$

cost of machine being Rs.10000 it costs more by ₹ 750 than it saves in terms of labour cost.

The present value of annual cost savings of the second machine =  
 $₹ 2200 \cdot P(5, 0.10) = ₹ 2200 \times 3.79079 = ₹ 8339.74$

Cost of the second machine being ₹ 8000, effective savings in labour cost is ₹ 339.74. Hence the second machine is preferable.

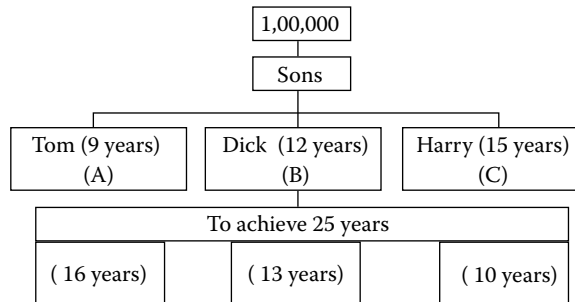
30. An investor intends purchasing a three year ₹ 1000 par value bond having nominal interest rate of 10%. At what price the bond may be purchased now if it matures at par and the investor requires a rate of return of 14%?

**Solution:** Present value of the bond =  $\frac{100}{(1+0.14)^1} + \frac{100}{(1+0.14)^2} +$   
 $\frac{100}{(1+0.14)^3} + \frac{1000}{(1+0.14)^3}$   
 $= 100 \times 0.87719 + 100 \times 0.769467 + 100 \times 0.674972$   
 $+ 1000 \times 0.674972$   
 $= 87.719 + 76.947 + 67.497 + 674.972 = 907.125$

Thus the purchase value of the bond is ₹ 907.125

31. Johnson left ₹ 1,00,000 with the direction that it should be divided in such a way that his minor sons Tom, Dick and Harry aged 9, 12 and 15 years should each receive equally after attaining the age 25 years. The rate of interest being 3.5%, how much each son receives after getting 25 years old?

**Solution:** Given problem can be explained as



$$A \left( 1 + \frac{3.5}{100} \right)^{16} = B \left( 1 + \frac{3.5}{100} \right)^{13} = C \left( 1 + \frac{3.5}{100} \right)^{10}$$

$$A(1.035)^{16} = B(1.035)^{13} = C(1.035)^{10}$$

$$A(1.035)^6 = B(1.035)^3 = C \text{-----}(I)$$

$$A : B : C = 1 : (1.035)^3 : (1.035)^6$$

$$A + B + C = 100000$$

$$x + x(1.035)^3 + x(1.035)^6 = 100000$$

$$x [1 + (1.035)^3 + (1.035)^6] = 10.0000$$

$$x(3.337973) = 1,00,000 \text{ then } x = ₹ 29958.30 \text{ (A' s share)}$$

$$B' s \text{ share} = 29958.30 (1.035)^3 = ₹ 33215.30$$

$$C' s \text{ share} = 29958.30 (1.035)^6 = ₹ 36826.40$$

32. A machine costs ₹ 5,20,000 with an estimated life of 25 years. A sinking fund is created to replace it by a new model at 25% higher cost after 25 years with a scrap value realization of ₹ 25000. what amount should be set aside every year if the sinking fund investments accumulate at 3.5% compound interest p.a.?

**Solution:** Cost of new machine =  $5,20,000 \times \frac{125}{100} = ₹ 6,50,000$ , Scrap value = ₹ 25,000

For new machine =  $650000 - 25000 = ₹ 6,25,000$ .

Here = ₹ 6,25,000, n = 25, i = 3.5% = 0.035

$$6,25,000 = P \cdot \left[ \frac{(1+i)^n - 1}{i} \right] = P \cdot \left[ \frac{(1+0.035)^{25} - 1}{0.035} \right]$$

$$6,25,000 = P \cdot [38.95] \text{ then } P = \frac{625000}{38.95} = ₹ 16046.27$$